Mode-dependent Differences in Chord Classification under an Original Computational Method of Tonal Structure Analysis

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***Abstract* — Basing upon original computational analytic method (Majchrzak 2005, 2007), the present work aims, at: 1) Showing the differences for the major key and the minor (harmonic) key in the classification of chords, as an aspect of importance for interpreting a piece's tonal structure diagram; 2) Drawing attention to the subordination of the minor key as versus the major key in the chord classification, using the same algorithm. The relations between chords appearing in the major and minor (harmonic) key are shown by applying the comparisons of: 1) third-based chords; 2) degrees in the C major and A minor keys, on which the same diatonic chords appear.**

Keywords: tonality, major key, minor key, analysis

I. INTRODUCTION

The invention of harmony in the baroque period was one source of polemics around music in 17th and 18th centuries. Among those who investigated the foundations of harmony on a philosophical basis, including the legitimacy of the two modes, are the founders of new scientific methods (Kepler 1619, Mersenne 1637, or, Descartes 1650). In the same period, both modes (i.e. major and minor) tend to be reduced to a single, i.e. major, scale – in that a minor is but a variety of the 'perfect' major scale. Theoretical works on scale modes, justifying the existence of scales, show minor scales as subordinate to the major. In Helmholtz's approach, the minor scale is not part of the music's beauty; nor can it be classed under the natural or rational system. Also Rameau (1722) was of opinion only the existence of the major mode is explainable in rational terms in the world of harmony. He considered the minor mode an unnatural variety of the major mode.

Our contemporary theoretical works on harmony, tonality, methods of main key determination in a musical piece, maps of chord relations, etc., are indicative of certain problems with the minor key (Shepard 1982, Chew 2000, Krumhansl 1990, Honingh 2007). The minor key issue also concerns the Author's computational method of analysis of the tonal structure in pieces of music (Majchrzak 2005, 2007).

II. METHOD OF ANALYSIS OF THE TONAL STRUCTURE

Original method consists in assignation of chords appearing in a piece of music to individual key ranges being keys in their respective natural variety. Using the analytical method in question, a diagram of tonal structure of a piece can be produced, such tonal structure being understood as quantitative relation of key ranges for which specific chords have been classified. We mark the keys with the consecutive integers: the sharp keys with positive numbers, the flat keys – with negative numbers. The absolute value of the integer designates the number of accidentals in the key. The number (3) marks the keys of A major and F sharp minor (natural); the number (-1) – the keys of F major and D minor (natural).

For any tone, we can determine the keys it appears in. For instance, the tone D appears in these keys: (-3, -2, -1, 0, 1, 2, 3)[1]. The tone E appears in the following keys: (-1, 0, 1, 2, 3, 4, 5)[2]. The tone C appears in these keys: (-5, -4, -3, -2, -1, 0, 1)[3]. This is similarly so for any and each chord. For example, the tones of the C major chord appears in the following keys: (-5, -4, -3, -2, -1, 0, 1), (-1, 0, 1, 2, 3, 4, 5), (-4, -3, -2, -1, 0, 1, 2).

Axioms:

1) In the event that one of the chord tones is an octave transposition of another, then, such a tone shall not be taken into account whilst classifying the chord;

2) Any tone being distant from one another by one or more octaves shall be approached on an equivalent basis.

The substratum for our chord classification is the arithmetic average of keys wherein the tones of a given diatonic chord appear:

$$\text{arithmetic average} = (x_1 + x_2 + x_3 + \dots + x_n) / n$$

$x_1 + x_2 + x_3 + \dots + x_n$ – keys wherein the tones of a given diatonic chord appear; n – number of all keys.

Examples:

1) DFsharp:

$$\text{AA (arithmetic average)} = 2 \quad \frac{(-3-2-1+0+1+2+3)+(1+2+3+4+5+6+7)}{7+7}$$

[1] (E flat major and C minor, B flat major and G minor, F major and D minor, C major and A minor, G major and E minor, D major and B minor, A major and F sharp minor).

[2] (F major and D minor, C major and A minor, G major and E minor, D major and B minor, A major and F sharp minor, E major and C sharp minor, B major and G sharp minor).

[3] (D flat major and B flat minor, A flat major and F minor, E flat major and C minor, B flat major and G minor, F major and D minor, C major and A minor, G major and E minor).

2) BDFA:

$$AA = 0{,}25$$

$$\frac{(0+1+2+3+4+5+6)+(-3+2+1+0+1+2+3)+(-6-5-4-3-2-1+0)+(-2-1+0+1+2+3+4)}{7+7+7+7}$$

3) DFsharpAC:

$$AA = 0{,}75$$

$$\frac{(-3-2-1+0+1+2+3)+(1+2+3+4+5+6+7)+(-2-1+0+1+2+3+4)+(-5-4-3-2-1+0+1)}{7+7+7+7}$$

4) GCsharp:

$$AA = 2$$

$$\frac{(-4-3-2-1+0+1+2)+(2+3+4+5+6+7+8)}{7+7}$$

In this method:

Arithmetic average space:

– all numeric values derivable from the above arithmetic-average formula. The arithmetic average space is divided into key ranges (KRs), each of which is a key range with a given number of clef signs. E.g., the key range of one-flat keys (F major and D minor) encompasses the arithmetic average space's open-ended range, spanning between -0,5 and -1,5. The key range of two-sharp keys (D major and B minor) encompasses the arithmetic average spanning between 1,5 and 2,5. The key range of four-sharp keys (E major and C# minor) encompasses the arithmetic average spanning between 4,5 and 5,5.

Chords and Key Range

Examples: the chord GBDF (AA = -0,25) belongs to KR 0. The chord EG#BC# (AA = 4) belongs to KR 4.

2KRs chord

– any chord whose arithmetic average belongs to two adjacent KRs. E.g., the arithmetic average of the CEGB chord is 0,5; the chord belongs to both KR 0 (C major and A minor) and KR 1 (G major and E minor). The arithmetic average of the CDEFGA chord is -0,5; the chord belongs to both KR -1 (F major and D minor) and KR 0 (C major and A minor).

N-D

– non-diatonic chords.

Analysis of pieces can be displayed in the form of diagram.

Example**:**

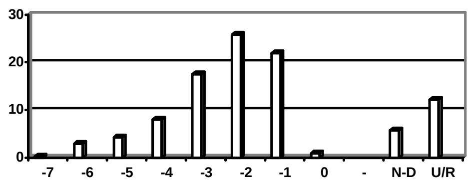

Fig. 1. Chopin, Mazurka B flat major, Op. 17, No. 1

Where:

Horizontal Axis:

Key ranges. For example: -4 (Key range of A flat major and F minor), -2 (Key range of B flat major and D minor), 0 (Key ranges of C major and A minor)

Vertical Axis:

Percentage domination of given key ranges

III. Third-based chords built on individual degrees of C major/A minor keys

As discussed hereinabove, the analytical method consists in assignment of diatonic chords to individual ranges of a key, which is followed by a quantitative comparison of the key ranges. Let us take a look at the differences in assignment to key ranges of triad appearing on individual grades of C major and A minor keys.

1) C major:

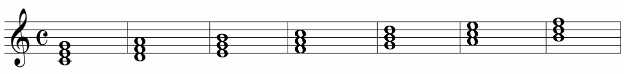

Fig. 2. Triads appearing on individual grades of C major key.

KR 0 KR -1 KR 1 KR -1 KR 1 KR 0 KR 0

2) A minor:

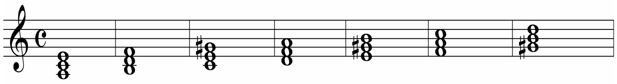

Fig. 2. Triads appearing on individual grades of A minor (harmonic).

KR 0 KR 0 N-D KR -1 KR 4 KR -1 KR 3

We could see above that all the triads built upon individual C major key grades are assigned to the key's three ranges, including: KR 0 (C major and A minor), KR -1 (F major and D minor), KR 1 (G major and E minor). Triads created on individual grades of the A minor harmonic key belong to the key's four ranges whilst one of them belongs to the N-D group. Now, let us have a closer look at third-based chords built up on individual C major and A minor (harmonic) key grades.

TABLE I.
Third-based chords built on individual degrees of C major and A minor (harmonic) keys.

Dyads				
	C major		A minor	
Degree	**Chord**	**Key range**	**Chord**	**Key range**
1st	CE	KR 0	AC	KR -1, KR 0
2nd	DF	KR -2, KR -1	BD	KR 1, KR 2
3th	EG	KR 0, KR 1	CE	KR 0
4th	FA	KR -1	DF	KR -2, KR -1
5th	GB	KR 1	EG#	KR 4
6th	AC	KR -1, KR0	FA	KR -1
7th	BD	KR 1, KR 2	G#B	KR 4, KR 5

Triads				
	C major		A minor	
Degree	**Chord**	**Key range**	**Chord**	**Key range**
1st	CEG	KR 0	ACE	KR 0
2nd	DFA	KR -1	BDF	KR 0
3th	EGB	KR 1	CEG#	N-D

4th	FAC	KR -1	DFA	KR -1
5th	GBD	KR 1	EG#B	KR 4
6th	ACE	KR 0	FAC	KR -1
7th	BDF	KR 0	G#BD	KR3

Four-note chords				
	C major		**A minor**	
Degree	**Chord**	**Key range**	**Key range**	**Key range**
1st	CEGB	KR 0, KR 1	ACEG#	N-D
2nd	DFAC	KR -1	BDFA	KR 0
3th	EGBD	KR 1	CEG#B	N-D
4th	FACE	KR -1, KR 0	DFAC	KR -1
5th	GBDF	KR 0	EG#BD	KR3
6th	ACEG	KR 0	FACE	KR -1, KR 0
7th	BDFA	KR 0	G#BDF	N-D

Five-note chords				
	C major		**A minor**	
Degree	**Chord**	**Key range**	**Key range**	**Key range**
1st	CEGBD	KR 0	ACEG#B	N-D
2nd	DFACE	KR 0	BDFAC	KR 0
3th	EGBDF	KR 0	CEG#BD	N-D
4th	FACEG	KR -1	DFACE	KR 0
5th	GBDFA	KR 0	EG#BDF	N-D
6th	ACEGB	KR 1	FACEG#	N-D
7th	BDFAC	KR 0	G#BDFA	N-D

Six-note chords				
	C major		**A minor**	
Degree	**Chord**	**Key range**	**Key range**	**Key range**
1st	CEGBDF	KR 0	ACEG#BD	N-D
2nd	DFACEG	KR -1, KR 0	BDFACEG#	N-D
3th	EGBDFA	KR 0	CEG#BD	N-D
4th	FACEGB	KR 0	DFACEG#	N-D
5th	GBDFAC	KR 0	EG#BDFA	N-D
6th	ACEGBD	KR 0, KR 1	FACEG#B	N-D
7th	BDFACE	KR 0	G#BDFAC	N-D

A. TRIADS

1st, 4th:

Chords based on 1st degree of C major and A minor keys are classed in the key range where they function as the keynotes, i.e. KR 0 (C major, A minor). The situation where triads built upon the same degree in the keys C major and A minor are part of a single KR is to be met only once: this concerns chords built on the fourth degree. The F major and D minor chords are part of PT -1. The situation is different for triads built on the remaining degrees of those keys.

2nd:

The chord built up on the 2nd degree of the C major – i.e. the subdominant of the 2nd degree belongs, as shown above, to KR -1, the range to which the C major subdominant chord belongs as well. In the A minor, the chord built on the 2nd degree is part of the same key range as the chord built on 1st degree (i.e. the minor keynote), that is, KR 0. Having said that, why should the triad built on the 2nd degree of A minor key belong to PT 0? The BDF chord may be considered as a dominant seventh without the root in C major key. It then appears in the key range within which the chord appears into which it is resolved (according to the classic theory of harmony, the BDF chord may be resolved to the C major chord).

3th:

The chord built up on the 3rd degree of the C major, composed of EGB tones, belongs to KR 1, and so, to the KR where the G major chord appears. The chord on the 3rd degree in the A minor is an augmented chord, which means that it is not assigned to a key range. Instead, it is classed under a separate group of non-diatonic chords (N-D).

5th:

The triad built on the 5th degree (the dominant) in the C major, i.e. the G major chord, is classified in KR 1. The range is situated right of the keynote's range (KR 0 in the C major). In A minor key, the minor keynote appears in KR 0. In turn, the chord built on the 5th degree of the A minor, i.e. the major dominant, is classified as KR 4. Then, how should the dominant's situation be explained, in a range fixed as many as four ranges away from the range where the minor keynote in the A minor is classed? The E major chord may act as a keynote for the E major key. Hence, it is contained within KR 4, similarly as the C major chord in KR 0 or the A flat major chord KR -4.

6th:

In the C major, the sixth-grade keynote (ACE) appears within the same key range as the keynote (CEG), i.e. KR 0. In the A minor, the triad built on the sixth degree is situated in KR -1, that is, a range located left of the range wherein the keynote chord appears.

7th:

In both C major and A major key, a diminished chord appears upon degree 7th. We have already come across the BDF chord on the grounds of A minor key (as its 2nd-degree chord). As for the GsharpBD chord appearing on the 7th degree of the A minor, the following question arises: How can we interpret the position of a chord built up on the 7th degree of A minor key in the range of A major and F sharp minor keys (KR 3)? The chord composed of the notes GsharpBD may be deemed to be the dominant seventh without the root for A major key, i.e. KR 3. Thus, the second of the diminished triads built upon the A minor degrees better corresponds with the major key (A major – KR 3) than with a minor one (A minor – KR 0).

B. FOUR-NOTE CHORDS

1st:

The arithmetic average of the keys where the notes appear of the four-part chord built on 1st degree of the C major (i.e. CEGB) equals 0,5. Thus, the chord belongs to both KR 0 and KR 1[4]. As for the minor key, one has to do with

[4] 2KRs.

a chord whose notes are not reducible to a single key of the natural variety (BDFGsharp)[5], and hence, we will not take it into consideration for the purpose of assignment to individual key ranges.

2^{nd}:

The four-note chord built on the 2^{nd} degree of the C major belongs to KR -1. Thich chord can be considered as the minor keynote with a small seventh added in the D minor, or, the keynote with a great sixth added in the F major. The four-part chord BDFA appearing on degree 2^{nd} of the A minor is classed under KR 0. Within this same range, C major chord appears, to which BDFA chord, being the C major key's dominant ninth without the root, gets most frequently resolved.

3^{rd}

The structure of four-note chord EGBD is identical to that of DFAC chord, whereas the CEGsharpB chord is put in our breakdown in a separate column (N-D), and not assigned to an individual key range.

4^{th}:

In the C major, the four-note chord built on degree 4^{th} has a structure identical as the four-note chord built on the key's 1^{st} degree (the chord belongs to both KR -1 and KR 0). In the A minor, the four-note chord built on the 4^{th} degree is of an identical structure as the one built on degree 1^{st} of the C major (the chord belongs to KR -1).

5^{th}:

The GBDF chord is part of KR 0, i.e. to that major-key range in which it operates as the dominant seventh. In the case of the A minor dominant seventh, we come across a troublesome case. This chord does not, namely, belong to the key range wherein the A minor keynote appears (i.e. KR 0), but is part of KR 3 instead. A similar situation was the case when it came to discussing the GsharpBD chord. The GBDF chord belongs to the range where the major tonic appears (CEG chord appearing in KR 0), whereas it is not part of the range where the major tonic appears to which this chord can be resolved (CEflatG appears in KR -3). A similar thing happens with EGsharpBD chord, which appears in the range where the major tonic is classed to which it is resolved, that is, in KR 3. (The A major chord to which EG#BD chord gets resolved appears in KR 3).

6^{th}, 7^{th}:

Chords build like: ACEG, FACE, BDFA have already been discussed. The diminished four-part chord is a non-diatonic chord (N-D).

C. Five-note chords

Most five-note chords built on A minor key degrees are part of the N-D group. In the C major, most of the chords belong to the key's main range, and two of them belong to KR -1 and KR 1, respectively.

D. Six-note chords

All the six-note chords created on individual degrees of the A minor, in its harmonic variety, contain an augmented four-part chord. This means that these are not assigned to key ranges (N-D). As for the C major, all the six-part chords belong to KR 0. Two of them, created upon degrees 2^{nd} and 6^{th}, respectively, are 2KRs chords.

[5] N-D.

E. Dyads

To end with, let us take a look on third-sized dyads. As it can be seen in the table above, the differences are remarkable also for the dyads.

IV. Diatonic chords of varied structure

The previous chapter discussed third-based chords built upon individual C major and A minor keys' degrees. Some of the minor-key chords were unclassified with respect to the key ranges, as their tones could not be reduced to a single natural key. Now, let us turn attention to diatonic chords with a diverse interval structure. The subsequent table lines specify chords belonging to KR 0 (C major, A minor) and the C major/A minor degrees whereupon the chords are created. Example: Let us take any triad, e.g. CDE. The arithmetic average equals 0, so the triad is contained within KR 0 (C major and A minor).

C major key degrees:				1	2	3	4	5	6	7	
Tones:	G	A	B	**C**	**D**	**E**	F	G	A	B	C
A minor key degrees:		1	2	3	4	5	6	7			

In the C major, it is built on degrees 1^{st}, 2^{nd} and 3^{rd}. These degrees appear more important than those in the case of A minor key (3^{rd}, 6^{th} 5^{th}). The first three notes of the C major comprise the tonic's prime and third, whereas in the A minor, these are the keynote's third and fifth with an added fourth.

As for 2KRs chords, two identically structured chords will be quoted in our tables: the first belonging to KR 0 and KR 1:

background:

and the second, to KR -1 and KR 0:
background:

TABLE II.
DIATONIC CHORDS WITH A DIVERSE INTERVAL STRUCTURE.

Dyads		
Chord	**C major key degrees**	**A minor key degrees**
G A B C D E F G A B C	**1, 3** (CE)	**3, 5** (CE)
G A B C D E F G A B C	**3, 5** (EG)	**5, 7** (EG)
G A B C D E F G A B C	**1, 6** (CA)	**1, 3** (AC)
G A B C D E F G A B C	**5, 6** (GA)	**1, 7** (AG)
G A B C D E F G A B C	**1, 7** (CB)	**2, 3** (BC)
G A B C D E F G A B C	**3, 4** (EF)	**5, 6** (EF)
G A B C D E F G A B C	**4, 7** (FB)	**2, 6** (BF)
G A B C D E F G A B C	**2, 6** (DA)	**1, 5** (AD)
G A B C D E F G A B C	**2, 5** (DG)	**4, 7** (DG)

Triads		
Chord	**C major key degrees**	**A minor key degrees**
G A B C D E F G A B C	**1, 3, 5** (CEG)	**3, 5, 7** (CEG)
G A B C D E F G A B C	**1, 3, 6** (CEA)	**1, 3, 5** (ACE)
G A B C D E F G A B C	**2, 5, 6** (DGA)	**1, 4, 7** (ADG)
G A B C D E F G A B C	**2, 4, 7** (DFB)	**2, 4, 6** (BDF)
G A B C D E F G A B C	**4, 6, 7** (FAB)	**1, 2, 6** (ABF)
G A B C D E F G A B C	**4, 5, 7** (FGB)	**2, 6, 7** (BFG)
G A Bb C D E F G A Bb C	**3, 6, 7b** (EABb)	**1, 2b, 5** (ABbE)
G A B C D E F# G A B C	**1, 4#, 5** (CF#G)	**3, 6#, 7** (CF#G)
G A B C D E F G A B C	**1, 2, 6** (CDA)	**1, 3, 4** (ACD)
G A B C D E F G A B C	**2, 3, 5** (DEG)	**4, 5, 7** (DEG)
G A B C D E F G A B C	**1, 5, 7** (CGB)	**2, 3, 7** (BCG)
G A B C D E F G A B C	**3, 4, 6** (EFA)	**1, 5, 6** (AEF)
G A B C D E F G A B C	**1, 2, 3** (CDE)	**3, 4, 5** (CDE)
G A B C D E F G A B C	**2, 3, 4** (DEF)	**4, 5, 6** (DEF)
G A B C D E F G A B C	**1, 2, 7** (CDB)	**2, 3, 4** (BCD)

Four-note chords		
Chord	**C major key degrees**	**A minor key degrees**
G A B C D E F G A B C	**2, 4, 5, 7** (DFGB)	**2, 4, 6, 7** (BDFG)
G A B C D E F G A B C	**2, 4, 6, 7** (DFAB)	**1, 2, 4, 6** (ABDF)
G A B C D E F G A B C	**2, 3, 5, 6** (DEGA)	**1, 4, 5, 7** (ADEG)
G A B C D E F G A B C	**1, 2, 5, 6** (CDGA)	**1, 3, 4, 7** (ACDG)
G A B C D E F G A B C	**1,3, 5, 6** (CEGA)	**1, 3, 5, 7** (ACEG)
G A B C D E F G A B C	**1, 3, 5, 7** (CEGB)	**2, 3, 5, 7** (BCEG)
G A B C D E F G A B C	**1, 3, 4, 6** (CEFA)	**1, 3, 5, 6** (ACEF)
G A B C D E F G A B C	**1, 2, 3, 6** (CDEA)	**1, 3, 4, 5** (ACDE)
G A B C D E F G A B C	**1, 2, 3, 5** (CDEG)	**3, 4, 5, 7** (CDEG)
G A B C D E F# G A B C	**1, 2, 4#, 5** (CDF#G)	**3, 4, 6#, 7** (CDF#G)
G A Bb C D E F G A Bb C	**2, 3, 6, 7b** (DEABb)	**1, 2b, 4, 5** (ABbDE)
G A B C D E F G A B C	**1, 4, 6, 7** (CFAB)	**1, 2, 3, 6** (ABCF)
G A B C D E F G A B C	**3, 4, 5, 7** (EFGB)	**2, 5, 6, 7** (BEFG)
G A B C D E F G A B C	**1, 2, 5, 7** (CDGB)	**2, 3, 4, 7** (BCDG)
G A B C D E F G A B C	**4, 5, 6, 7** (FGAB)	**1, 2, 6, 7** (ABFG)
G A B C D E F G A B C	**1, 3, 4, 7** (CEFB)	**2, 3, 5, 6** (BCEF)
G A B C D E F G A B C	**2, 3, 4, 7** (DEFB)	**2, 4, 5, 6** (BDEF)
G A Bb C D E F G A Bb C	**3, 5, 6, 7b** (EGABb)	**1, 2b, 5, 7** (ABbEG)
G A B C D E F# G A B C	**1, 4#, 5, 6** (CF#GA)	**1, 3, 6#, 7** (ACF#G)
G A B C D E F G A B C	**1, 2, 4, 7** (CDFB)	**2, 3, 4, 6** (BCDF)
G A B C D E F G A B C	**1, 5, 6, 7** (CGAB)	**1, 2, 3, 7** (ABCG)

G	A	B	C	D	E	F	G	A	B	C	**1, 2, 6, 7** (CDAB)	**1, 2, 3, 4** (ABCD)
G	A	B	C	D	E	F	G	A	B	C	**2, 3, 4, 5** (DEFG)	**4, 5, 6, 7** (DEFG)

Five-note chords												
Chord											**C major key degrees**	**A minor key degrees**
G	A	B	C	D	E	F	G	A	B	C	**1, 2, 3, 5, 6** (CDEGA)	**1, 3, 4, 5, 7** (ACDEG)
G	A	B	C	D	E	F	G	A	B	C	**2, 4, 5, 6, 7** (DFGAB)	**1, 2, 4, 6, 7** (ABDFG)
G	A	Bb	C	D	E	F	G	A	Bb	C	**2, 3, 5, 6, 7b** (DEGABb)	**1, 2b, 4, 5, 7** (ABbDEG)
G	A	B	C	D	E	F#	G	A	B	C	**1, 2, 4#, 5, 6** (CDF#GA)	**1, 3, 4, 6#, 7** (ACDF#G)
G	A	B	C	D	E	F	G	A	B	C	**1, 2, 3, 4, 6** (CDEFA)	**1, 3, 4, 5, 6** (ACDEF)
G	A	B	C	D	E	F	G	A	B	C	**1, 2, 3, 5, 7** (CDEGB)	**2, 3, 4, 5, 7** (BCDEG)
G	A	B	C	D	E	F	G	A	B	C	**1, 3, 4, 6, 7** (CEFAB)	**1, 2, 3, 5, 6** (ABCEF)
G	A	B	C	D	E	F	G	A	B	C	**1, 3, 4, 5, 7** (CEFGB)	**2, 3, 5, 6, 7** (BCEFG)
G	A	B	C	D	E	F	G	A	B	C	**2, 3, 4, 5, 7** (DEFGB)	**2, 4, 5, 6, 7** (BDEFG)
G	A	B	C	D	E	F	G	A	B	C	**1, 2, 4, 6, 7** (CDFAB)	**1, 2, 3, 4, 6** (ABCDF)
G	A	B	C	D	E	F	G	A	B	C	**1, 4, 5, 6, 7** (CFGAB)	**1, 2, 3, 6, 7** (ABCFG)
G	A	B	C	D	E	F	G	A	B	C	**3, 4, 5, 6, 7** (EFGAB)	**1, 2, 5, 6, 7** (ABEFG)
G	A	B	C	D	E	F	G	A	B	C	**1, 2, 3, 4, 7** (CDEFB)	**2, 3, 4, 5, 6** (BCDEF)
G	A	B	C	D	E	F	G	A	B	C	**2, 3, 4, 5, 6** (DEFGA)	**1, 4, 5, 6, 7** (ADEFG)
G	A	B	C	D	E	F	G	A	B	C	**1, 3, 4, 5, 7** (CEFGB)	**2, 3, 5, 6, 7** (BCEFG)

Six-note chords												
Chord											**C major key degrees**	**A minor key degrees**
G	A	B	C	D	E	F	G	A	B	C	**1, 2, 4, 5, 6, 7** (CDFGAB)	**1, 2, 3, 4, 6, 7** (ABCDFG)
G	A	B	C	D	E	F	G	A	B	C	**2, 3, 4, 5, 6, 7** (DEFGAB)	**1, 2, 4, 5, 6, 7** (ABDEFG)
G	A	B	C	D	E	F	G	A	B	C	**1, 2, 3, 5, 6, 7** (CDEGAB)	**1, 2, 3, 4, 5, 7** (ABCDEG)
G	A	B	C	D	E	F	G	A	B	C	**1, 2, 3, 4, 5, 6** (CDEFGA)	**1, 3, 4, 5, 6, 7** (ACDEFG)
G	A	B	C	D	E	F	G	A	B	C	**1, 3, 4, 5, 6, 7** (CEFGAB)	**1, 2, 3, 5, 6, 7** (ABCEFG)
G	A	B	C	D	E	F	G	A	B	C	**1, 2, 3, 4, 6, 7** (CDEFAB)	**1, 2, 3, 4, 5, 6** (ABCDEF)
G	A	B	C	D	E	F	G	A	B	C	**1, 2, 3, 4, 5, 7** (CDEFGB)	**2, 3, 4, 5, 6, 7** (BCDEFG)

V. CONCLUSIONS

The examples discussed above enable us to draw attention to the differences in the assignment of chords to the key ranges, conditional upon the key's mode.

Basing upon the examples quoted (C major and A minor keys, in our case), tentative conclusions may be drawn with respect to a superiority of the major key over the minor: 1) In major key, dominant forms are frequently contained within the key range in which the major tonic appears to which they are resolvable. In the minor, dominant forms are distant from the range wherein the minor chord (minor keynote) appears to which they are resolved. 2) In minor key (harmonic variety), chords appear that are not assignable to key ranges (which also refers to dominant forms, e.g. dominant ninth with a small ninth, or, diminished four-part chord).

ACKNOWLEDGMENT

I am grateful to Dr Alan Marsden, Lancaster University, for helpful comments. I thank also Tristan Korecki for translation assistance.

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